Problem 59

$f(x)$ is an increasing function satisfying the condition that $f(x+10) = f(x) + 4$ for all real numbers x.
If $f(0) = 0, f(1) = 1, f(3) = 2, f(6) = 3, f(10) = 4$ what is x when $f(x) = 2015$?

Answer

$\boxed{5036}$

Explanation

$f(x)$ is 'periodic' (actually called arithmetic quasiperiodic) in the sense that shifting the graph 4 units in Δy and 10 units in Δx results in the same graph. Since $(0, 0)$ lies on the function, and $2015 = 503(4) + 3$; there are 503 cycles with a remainder of 3. So, $f^{-1}(2015) = 503(10) + f^{-1}(3) \Rightarrow 5030 + 6 \Rightarrow 5036$