Problem 53

There are 600 days on planet Gamma. Two moons, Alpha & Beta orbit planet Gamma at the rates of 1 revolution every 120 and 75 days respectively. If the orbits of Alpha and Beta are concentric circles which lie in the same plane with Alpha's orbit being the circle of smaller radius, Alpha eclipses Beta every how many days?

Answer

$\boxed{200 \text{ days}}$

Explanation

Let x_α, x_β be the distance (in revolutions) of Alpha & Beta respectively, and t is in years.

120 and 75 days imply 5 and 8 revolutions per year. Let $x_\alpha = 5t$
$x_\beta = 8t$

If at $t = 0$ an eclipse happens, the next eclipse would happen where $x_\beta - x_\alpha = 1$. Thus, $8t - 5t = 1 \Rightarrow t = \frac{1}{3} \Rightarrow t = 200$ days