Problem 51

If a, b are such that there exists a k which makes $f(x)$ continuous at a, find k.

$$f(x) \begin{cases} b\sqrt{x} - k & x < a \\ \pi & x = a \\ \frac{x^2 - k^2}{x - a} & x > a \end{cases}$$

Answer

$$\boxed{-\dfrac{\pi}{2}}$$

Explanation

If $f(x)$ is continuous, then we have:

$$\lim_{x \to a^-} b\sqrt{x-k} \;=\; \pi \;=\; \lim_{x \to a^+} \frac{x^2 - k^2}{x - a}$$

The third expression implies that $k = \pm a$. However, if $k = a$, the first limit does not exist for any values of b. Thus, $k = -a$. Evaluating the limits you find that $a = \frac{\pi}{2}$ and $b = \sqrt{\pi}$, and $k = -a \Rightarrow k = -\frac{\pi}{2}$