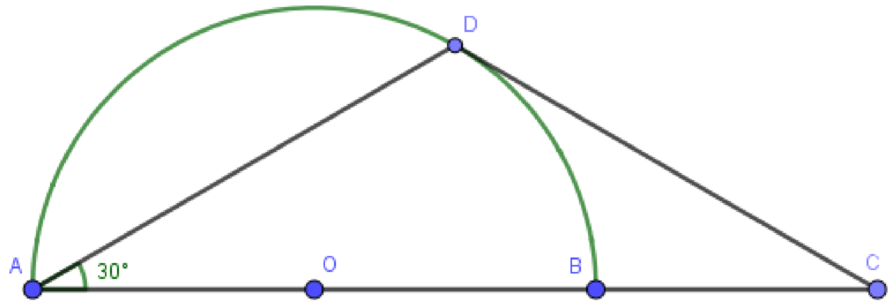
Problem 48

Consider semicircle O with radius r. Diameter $\overline{AOB}$ is extended to point C. Chords $\overline{AD}$ and $\overline{CD}$ are drawn so that $m\angle DAO = 30^\circ$ and $\overline{CD}$ is tangent to semicircle O. Find the area of triangle ADC in terms r.



Answer

$$\boxed{\frac{3\sqrt{3}\ r^2}{4}}$$

Explanation

Construct line segment $\overline{BD}$. Since inscribed triangle ABD shares a side with the diameter which has a length of $2r$, we discover that ABD is a $30-60-90$ right triangle with $AD = \sqrt{3}r$. Also, since the measure of inscribed angle A is $30°$, so is the measure of $\widehat{BD}$. Therefore, the measure of $\widehat{DA}$ is $60°$, and the measure of external angle C is also $30°$ making triangle ACD isosceles with vertex angle measuring $120°$. Thus the area is $A = \frac{1}{2}l^2 \sin(120\,\text{deg}) = (\frac{1}{2})(\sqrt{3}r)^2)\frac{\sqrt{3}}{2}) = \dfrac{3\sqrt{3}\ r^2}{4}$