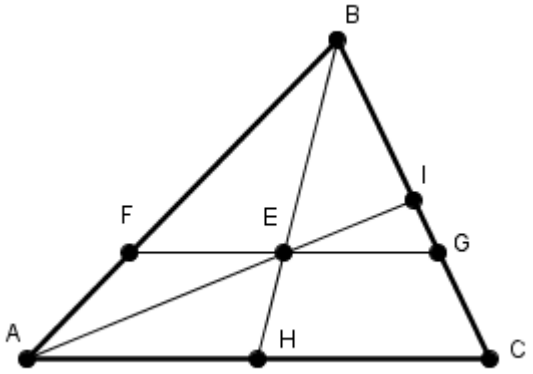
Problem 47

Consider isosceles triange BAC with vertex angle A. Median $\overline{BEH}$ and angle bisector $\overline{AEI}$ are drawn. $\overline{FEG}$ is then drawn parallel to side $\overline{AC}$. Find the ratio of the area of triangle FBG to the area of triangle ABC.



Answer

$\boxed{\frac{4}{9}}$

Explanation

Since the triangle is isosceles, angle bisector $\overline{AEI}$ is also a median. Hence E is the centroid of triangle ABC. This implies that the ratio of $\overline{BE}$ to $\overline{BH}$ is $2:3$. Furthermore, triangle FBG and ABC are similar with corresponding parts $\overline{BE}$, $\overline{BH}$. Thus the ratio of the areas would be $(\frac{2}{3})^2 = \frac{4}{9}$