Problem 45

Write a simplified equation in the form of $y = f(x)$, which describes the locus of centers of all circles which pass through the point $(0, \frac{1}{2})$ and are tangent to the x-axis.

Answer

$$\boxed{y = x^2 + \frac{1}{4}}$$

Explanation

The locus of points is a parabola with focus $(0, \frac{1}{2})$ and directrix $y = 0$. So, the equation is $y = x^2 + \frac{1}{4}$