# Problem 40

The median age of 10 women is greater than the median age of 6 men. At most how many man-woman pairs result in the woman being younger than the man?

# Answer

$\boxed{42}$

## Explanation

Consider 4 age groups.

4 women aged 20
3 men aged 21
6 women aged 24
3 men aged 25

Since there are 10 women, the median is the average of the $5^{\text{th}}$ and $6^{\text{th}}$ women's ages. Thus, it is 24.

Since there are 6 men, the median is the average of the $3^{\text{rd}}$ and $4^{\text{th}}$ men's ages. Thus, it is $\frac{21}{25} = 23$.

So, this distribution holds for the information.

Since the 4 youngest women are younger than all the men, there are $4 \times 6 = 24$ of these. The 6 eldest women are also younger than the 3 eldest men, so the number of pairs there is $6 \times 3 = 18$. So, there are $24 + 18 = 42$ pairs.

For some intuition as to why this result is maximal, consider moving a woman from the eldest women's group to the youngest one. Now, in order to keep the median to still be larger than the men, we would need to either make the youngest men younger than the women, or move a man from the eldest group to the youngest one as well. Doing so, results in only 38 pairs at most.