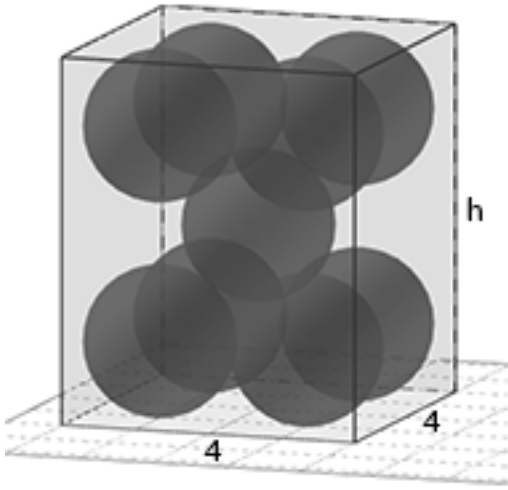


Problem 38

Consider 9 identical spheres of radius 1 are "packed" inside of a rectangular prism with a square base whose sides are of length 4. Each side of the prism is tangent to 4 spheres, and the sphere in the center is tangent to the other 8 (*see picture*). What is the height of the prism?



Answer

$$\boxed{2(1 + \sqrt{2})}$$

Explanation

Consider placing the prism in a coordinate system with $(0, 0, 0)$ being the coordinates of the front-left vertex of the prism.

The coordinates of the center of front-left sphere is $(1, 1, 1)$

The coordinates of the center of middle sphere is $(2, 2, \frac{h}{2})$

Since the two spheres are tangent to each other, the distance between their centers would be twice the radius, 2.

Thus,

$$\sqrt{(2-1)^2 + (2-1)^2 + (1-\frac{h}{2})^2} = 2$$

$$3 - h + \frac{h^2}{4} = 4$$

$$h^2 - 4h - 4 = 0$$

$$h = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2(1)}$$

$$h = 2(1 \pm \sqrt{2})$$

Since $h > 0$, we get the follow result:

$$h = 2(1 + \sqrt{2})$$