

Problem 35

What is the volume of the region enclosed by the locus of points in space 1 unit away from a line segment 1 unit long?

Answer

$$\boxed{\frac{7\pi}{3}}$$

Explanation

This region is a 'pill-shaped'. In other words it is a cylinder with height and radius of 1 and capped on both ends by semi-circles.

Hence:

$$V_{\text{region}} = V_{\text{circle}} + V_{\text{cyl}} \Rightarrow V_{\text{region}} = \frac{4}{3}\pi r^3 + \pi r^2$$

$$\Rightarrow V_{\text{region}} = \frac{4\pi}{3} + \pi = \frac{7\pi}{3}$$