

Problem 33

4 balls are taken at random from an infinitely large bin containing equal numbers of black and white balls only, and placed in a jar. A single ball is now drawn from the jar. After the ball is drawn, we observe that there are 2 black balls and 1 white ball left in the jar. What is the probability the ball that was drawn from the jar was black?

Answer

$$\boxed{\frac{1}{2}}$$

Explanation

The probability the jar contained 3 black and 1 white is ${}_4C_3\left(\frac{1}{2}\right)^4 \rightarrow \frac{1}{4}$. The probability the jar contained 2 black and 2 white is ${}_4C_2\left(\frac{1}{2}\right)^4 \rightarrow \frac{3}{8}$. If the jar contained $3b, 1w$, the probability of drawing a black is $\frac{3}{4}$; If the jar contained $2b, 2w$, the probability of drawing a white is $\frac{1}{2}$. So, by Bayes' Theorem, the probability the ball drawn is black is $\frac{\frac{1}{4}\frac{3}{4}}{\frac{1}{4}\frac{3}{4} + \frac{3}{8}\frac{1}{2}} \Rightarrow \frac{1}{2}$