Problem 28

The distance a 3-dimensional point is from the origin is given by:

$$D_{(x,y,z)} = \sqrt{x^2 + y^2 + z^2}$$

If $\begin{cases} x_1 &=& r\cos A \sin B \\ y_1 &=& r\sin A \sin B \\ z_1 &=& r\cos B \end{cases}$, $r > 0$

What is $D_{(x_1,y_1,z_1)}$?

Answer

$\boxed{r}$

Explanation

Plugging (x_1, y_1, z_1) into $D_{(x,y,z)}$ yields:

$$\sqrt{(r\cos A \sin B)^2 + (r\sin A \sin B)^2 + (r\cos B)^2}$$

$$\Rightarrow \sqrt{r^2 \cos^2 A \sin^2 B + r^2 \sin^2 A \sin^2 B + r^2 \cos^2 B}$$

$$\Rightarrow \sqrt{r^2((\cos^2 A + \sin^2 A)\sin^2 B + \cos^2 B)}$$

$$\Rightarrow \sqrt{r^2((1)\sin^2 B + \cos^2 B)}$$

$$\Rightarrow \sqrt{r^2(\sin^2 B + \cos^2 B)}$$

$$\Rightarrow \sqrt{r^2(1)}$$

$$\Rightarrow \sqrt{r^2}$$

$$\Rightarrow |r| = r$$