

## Problem 27 (Calc)

If  $R_f = \left[ \frac{\sqrt{5}}{5}, 2 \right]$ ; and  $g(x) = x + \frac{1}{x}$

What is the range of  $(g \circ f)(x)$ ?

## Answer

$$\left[2, \frac{6\sqrt{5}}{5}\right]$$

## Explanation

On  $\left[\frac{\sqrt{5}}{5}, 2\right]$ ,  $g(x)$  has a local and absolute minimum at  $(1, 2)$  and no local maxima. Thus, the absolute maxima must occur at one of the endpoints.

$$\text{Since } f\left(\frac{\sqrt{5}}{5}\right) = \sqrt{5} + \frac{1}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$$

$$\text{and } f(2) = 2 + \frac{1}{2} = \frac{5}{2} < \frac{6\sqrt{5}}{5}$$

The maximum is  $\left(\frac{\sqrt{5}}{5}, \frac{6\sqrt{5}}{5}\right)$

Furthermore, since  $g(x)$  is continuous on  $\left[\frac{\sqrt{5}}{5}, 2\right]$ , the range is  $\left[2, \frac{6\sqrt{5}}{5}\right]$