Problem 24

Let $\{a_i\}_{i\geq 1}$ be a positive geometric sequence with common ratio r and k^{th} term of 1. Let $\{b_i\}_{i\geq 1}$ be an arithmetic sequence with common difference d. If $\{b_i\}_{i\geq 1} = \{\log(a_i)\}_{i\geq 1}$, what is $\dfrac{b_1}{d}$?

Answer

$\boxed{1-k}$

Explanation

If the k^{th} term of $\{a_i\}_{i\geq 1}$ is 1, then the closed form in terms of n is:

$a_n = r^{1-k} r^{n-1}$

Thus, the closed from of $\{b_i\}_{i\geq 1}$ in terms of n is:

$b_n = \log\left(a_n\right) \Rightarrow b_n = \log\left(r^{1-k} r^{n-1}\right) \Rightarrow b_n = \log\left(r^{1-k} r^{n-1}\right)$

$\Rightarrow b_n = (1-k)\log(r) + (n-1)\log(r)$

Thus $b_1 = (1-k)\log(r)$ and $d = \log(r)$, so

$\frac{b_1}{d} = \frac{(1-k)\log(r)}{\log(r)} = 1-k$