Problem 21

Consider the parabola, $y = ax^2 + bx + c$. If $(-1, 9)$ and $(2, 9)$ lie on the parabola, and the parabola is also tangent to the x-axis, write (a, b, c) as an ordered triple.

Answer

$\boxed{(4, -4, 1)}$

Explanation

Since $f(-1) = f(2)$, the axis of symmetry is $x = \frac{1}{2}$. Thus, the vertex is $(\frac{1}{2}, 0)$. Writing $f(x)$ in vertex form, we have: $f(x) = a(x - \frac{1}{2})^2$. Substituting $(2, 9)$, we have: $9 = a(2 - \frac{1}{2})^2 \Rightarrow 9 = \frac{9}{4}a \Rightarrow a = 4 \Rightarrow 4(x - \frac{1}{2})^2 \Rightarrow 4x^2 - 4x + 1$