Problem 18

How many solutions on $0 \leq \theta < 2\pi$ are there for the following equation?

$$\left(4\sin^2(\theta)\right)\left(4\sin^2(\theta)-1\right)\left(4\sin^2(\theta)-2\right)\left(4\sin^2(\theta)-3\right)\left(4\sin^2(\theta)-4\right)=0$$

Answer

$\boxed{16}$

Explanation

Solving for $\sin(\theta)$, we get:

$\sin(\theta) = \left\{0, \pm\frac{1}{2}, \pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{3}}{2}, \pm 1\right\}$

$\Rightarrow \theta = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}\right\}$

These are all the solutions around the typical Unit Circle.

So, there are 16 solutions.