Problem 13 (Calc)

If $a, b \in \mathbb{R}$, and $[a, b]$ contains 2014 integers, what is the maximum # of integers which can be contained in $[2014a, 2014b]$?

Answer

$4,058,209$

Explanation

Without loss of generalization, let us say that the integers contained in $[a, b]$ are $\{1, 2, \ldots, 2014\}$

This implies that $a \in (0, 1]$ and $b \in [2014, 2015)$.

Since we are multiplying the endpoints by 2014, we should make this interval as large as possible within: $(0, 2015)$. Thus, $[2014a, 2014b]$ can be arbitrarily close to $(0, 2014 * 2015)$ which contains $2014 * 2015 - 1$ integers.

$2014 * 2015 - 1 = 4,058,209$