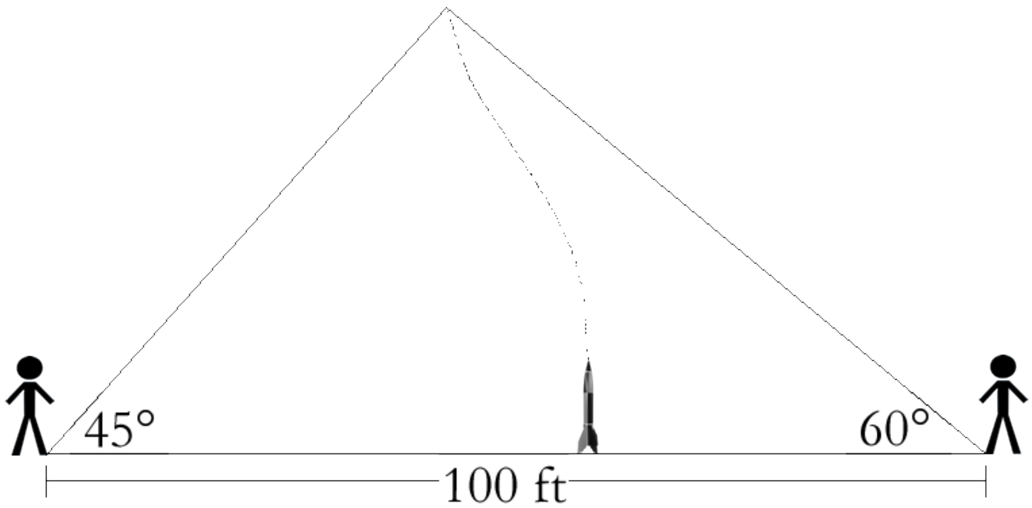
Problem 10



A model rocket is launched at noon on a windy day. 2 people stand 100 ft away from each other, on opposing sides of the rocket. Both people can measure the angle of elevation from the ground where they are standing to the rocket. If the shadow of the rocket (assumed to be directly below it) lies somewhere between the observers when it reaches its apex, and the angles of elevation are $60°$ and $45°$ respectively, what was the highest altitude the rocket reached?

Answer

$\boxed{50(3-\sqrt{3})\text{ ft}}$

Explanation

Construct the altitude from the apex of the rocket's flight to the ground. Let x be the length from person_1 to the altitude, and h be the length of the altitude.

Since the distance between person_1 and person_2 is 100 ft we have the distance from the altitude to person_2 is $100 - x$.

Using Right Triangle Trigonometry, we can construct the following system:

$$\begin{cases} \tan(45^\circ) = \dfrac{h}{x} \\ \tan(60^\circ) = \dfrac{h}{100-x} \end{cases}$$

For the first equation; $\tan(45^\circ) = 1$, so $x = h$.

Plugging into the second equation yields:

$$\tan(60^\circ) = \frac{h}{100-h}$$

$$\Rightarrow \sqrt{3}(100-h) = h$$

$$\Rightarrow 100\sqrt{3} - h\sqrt{3} = h$$

$$\Rightarrow 100\sqrt{3} = h\sqrt{3} + h$$

$$\Rightarrow 100\sqrt{3} = h(\sqrt{3}+1)$$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}+1}$$

$$\Rightarrow h = \frac{100\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)((\sqrt{3}-1))}$$

$$\Rightarrow h = \frac{100(3-\sqrt{3})}{3-1} \Rightarrow h = \frac{100(3-\sqrt{3})}{2} \Rightarrow 50(3-\sqrt{3})\text{ ft}$$