

Problem 9

A train travels non-stop from New York to Denver. If the speed is set to 5 mph faster than normal, the train will arrive 5 hours early. If the speed is set to 10 mph faster than normal, they will arrive 9 hours early. How much earlier will they arrive if the speed is set to 20 mph faster than normal?

Answer

15 hours

Explanation

Using $d = rt$, and plugging in the data from the second and third sentences, we have the following system:

$$\begin{cases} d = rt & (1) \\ d = (r + 5)(t - 5) & (2) \\ d = (r + 10)(t - 9) & (3) \end{cases}$$

Setting equations (1) equal to (2), we have:

$$rt = (r + 5)(t - 5) \Rightarrow -5r + 5t = 25$$

Setting equations (1) equal to (3), we have:

$$rt = (r + 10)(t - 9) \Rightarrow -9r + 10t = 90$$

$$\begin{cases} -5r + 5t = 25 \\ -9r + 10t = 90 \end{cases} \Rightarrow \begin{cases} 10r - 10t = -50 \\ -9r + 10t = 90 \end{cases} \Rightarrow r = 40$$

$$\Rightarrow -5(40) + 5t = 25 \Rightarrow 5t = 225 \Rightarrow t = 45$$

$$\Rightarrow d = (40)(45) \Rightarrow d = 1800$$

Therefore setting the speed to 20 mph faster than normal is 60 mph. Plugging in, we have:

$$d = rt \Rightarrow 1800 = (60)t \Rightarrow t = 30 \text{ hours}$$

Since the normal time of arrival is 45 hours, the train arrives 15 hours earlier than normal.