

Problem 8

If $\sqrt{a+b} = \sqrt[3]{a-b} = c \neq 1$, $a > b$

What is $\log_c(a^2 - b^2)$?

Answer

5

Explanation

The definition of the $\log(x)$ yields:

$$\sqrt{a+b} = c \Rightarrow \log_{a+b}(c) = \frac{1}{2}$$

$$\sqrt[3]{a-b} = c \Rightarrow \log_{a-b}(c) = \frac{1}{3}$$

Using the the difference of two squares, properties of $\log(x)$, and the change of base formula, we have:

$$\log_c(a^2 - b^2)$$

$$\log_c[(a+b)(a-b)]$$

$$\Rightarrow \log_c(a+b) + \log_c(a-b)$$

$$\Rightarrow \frac{\log_{a+b}(a+b)}{\log_{a+b}(c)} + \frac{\log_{a-b}(a-b)}{\log_{a-b}(c)}$$

$$\Rightarrow \frac{1}{1/2} + \frac{1}{1/3}$$

$$\Rightarrow 2 + 3 \Rightarrow 5$$