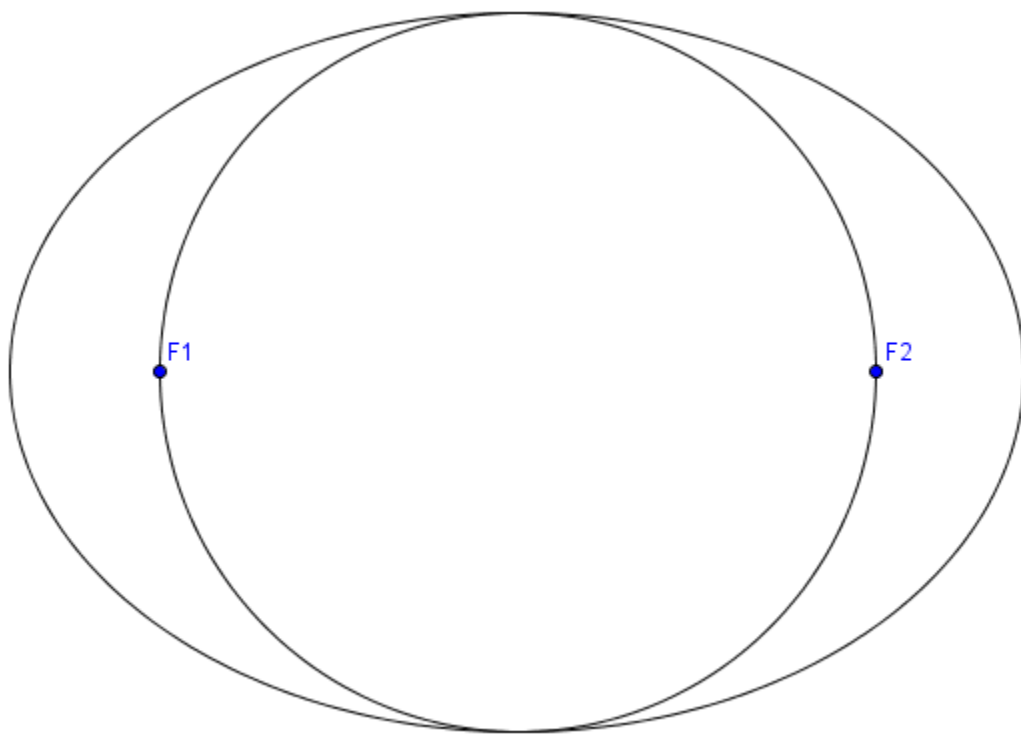


Problem 7

Consider the ellipse which circumscribes a circle such that the foci of the ellipse lie on the circle.



What is the ratio of the area of the ellipse to the area of the circle?

Note: $A_{\text{ellipse}} = \pi ab$, where a, b are the radii of the ellipse along the major and minor axes respectively.

Answer

$$\boxed{\sqrt{2}}$$

Explanation

Let the radius of the circle be r . Clearly the radius along the minor axis, b , is equal to r . Since the foci lie on the circle, the distance from the foci and the center of the ellipse, c , would also be equal to r .

Using the formula: $c^2 = a^2 - b^2$, and plugging in $b = c = r$, we have:

$$r^2 = a^2 - r^2$$

$$a^2 = 2r^2$$

$$a = \sqrt{2}r$$

Thus:

$$A_{\text{ellipse}} = \pi ab = \sqrt{2}\pi r^2 \text{ and}$$

$$A_{\text{circle}} = \pi r^2$$

$$\text{So, } \frac{A_{\text{ellipse}}}{A_{\text{circle}}} = \frac{\sqrt{2}\pi r^2}{\pi r^2} = \sqrt{2}$$