

Problem 4

Let $f(x) = \frac{4x}{x-2}$ and consider the sequence recursively defined by: $x_{n+1} = f(x_n)$

If $x_2 = 10$, find the sum of the first 4 terms.

Answer

25

Explanation

$$x_1 = f^{-1}(10) \rightarrow 10 = \frac{4x}{x-2} \rightarrow x_1 = \frac{20}{6} = \frac{10}{3}$$

$$x_2 = 10$$

$$x_3 = f(10) = \frac{4(10)}{(10)-2} = \frac{40}{8} = 5$$

$$x_4 = f(f(10)) = f(5) = \frac{4(5)}{(5)-2} = \frac{20}{3}$$

$$\text{Thus, } x_1 + x_2 + x_3 + x_4 = \frac{10}{3} + 10 + 5 + \frac{20}{3} = 25$$