Problem 2

If $\sin(2x) = \frac{4}{5}$ and $0° \leq x \leq 45°$, what is the exact value of

$\cos^6(x) - \sin^6(x)$?

Answer

$$\boxed{\frac{63}{125}}$$

Explanation

Since $\sin(2x) = \frac{4}{5}$, we have by use of the Pythagorean Identity, $\cos(2x) = \frac{3}{5}$

Now,

$\cos^6(x) - \sin^6(x)$

$(\cos^3(x) + \sin^3(x))(\cos^3(x) - \sin^3(x))$, Difference of Two Squares

$(\cos(x) + \sin(x))(\cos^2(x) - \sin(x)\cos(x) + \sin^2(x))\times$
$(\cos(x) - \sin(x))(\cos^2(x) + \sin(x)\cos(x) + \sin^2(x))$, Sum and Difference of Two Cubes

$(\cos^2(x) - \sin^2(x))(1 - \sin(x)\cos(x))(1 + \sin(x)\cos(x))$, Pythagorean Identity and Difference of Two Squares

$\cos(2x)(1 - \frac{1}{2}\sin(2x))(1 + \frac{1}{2}\sin(2x))$, Double Angle Identities

$\frac{3}{5}(1 - \frac{2}{5})(1 + \frac{2}{5})$, Plugging in

$(\frac{3}{5})(\frac{3}{5})(\frac{7}{5}) = \frac{63}{125}$